Calibration Sphere Error Simulation

Table of Contents

Introduction

This article describes the effect of form error on a sphere when used as a calibration artifact on a CMM. The purpose of the calibration sphere is to determine the effective stylus tip diameter and relative probe offset between multiple probe positions. Multiple probe positions can be from a fixed head with more than one stylus or a single stylus on an articulating probe head.

The method used to test the influence of form error from the CMM calibration sphere is to simulate the measurement of an imperfect sphere and report the error in position and size of the tip stylus. Using a simulation allows almost infinite possibilities when testing for this kind of measurement error and is almost impossible to do by other means.

The calibration sphere simulation program is a console utility that is suitable for testing various measurement conditions. The data used for this article was created by generating a shell script file containing all the desired test conditions.

Sphere Measurement Simulation

There are three known methods that could have been used to simulate the sphere measurement with different amounts of form error:

1. Randomize the amount of error on each target measurement point.
2. Deform the shape of the sphere in a non-symmetrical way such as adding a flat spot to one side.
3. Deform the shape of the sphere by stretching and compressing along specific axis.

Method (1) is suited for cases where measurement noise such as repeatability or scale interpolation errors are part of the sphere measurement. There is a problem estimating the real sphere form error as only the measurement points are available for analysis and not the entire sphere shape so this method is not well suited for this purpose.

Method (2) where the shape of the sphere is deformed by adding a flat spot or by shifting one half of the sphere from the second half works but it is possible that patterns of points may completely miss the deformed area(s). The real form error of the sphere can be estimated by measuring a large sample of points. Creating a sphere with a shape that has the exact target form error is difficult and subject to interpretation. This method doesn't appear to be well suited for this purpose.

Method (3) where the shape of the sphere is deformed by stretching or compressing the sphere along orthogonal axis doesn't have the problems of the first two methods. The real form error can be measured using only six key points and it is impossible to measure points that are not affected by the deformation. Imperfect spheres are expected to have a shape typical of stretching or compressing along one or more axis where spheres that have been damaged are likely to have errors similar to method (2). Method (3) was selected for the simulation program.

Calibration Sphere Error Simulation

Sphere Generation Method

The method used to generate the theoretical measurement sphere is based on the following steps:

- Construct two random vectors.
- Calculate the cross product and make orthogonal.
- Scale up one of the vectors by half the form error and scale down one of the remaining vectors by half of the form error.

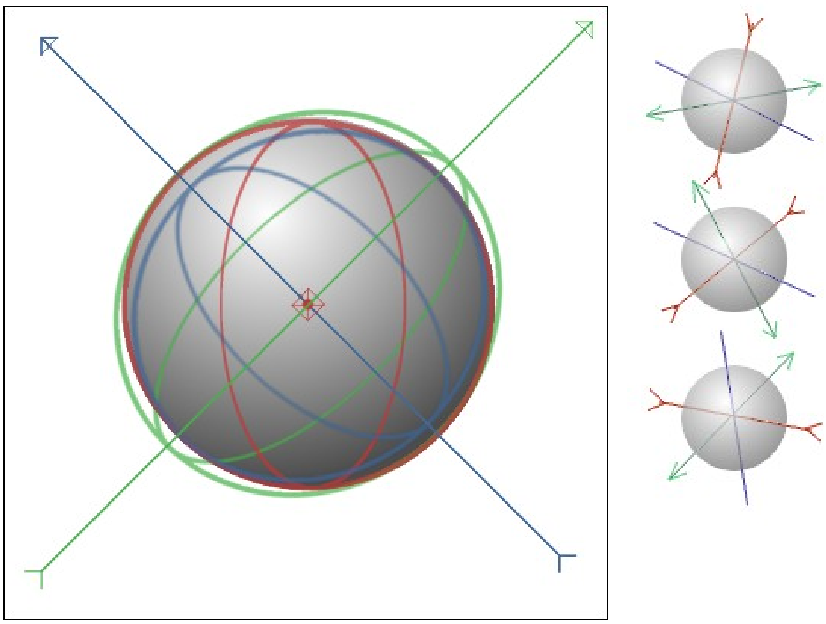

Figure 1: Sphere deformation by stretching or compressing along orthogonal axis.

One random sphere is generated for each set of simulated probe calibrations.

Probe Tip Calibration

The probe configuration for testing was a single tip on an articulating probe head with nine different probe orientations. Testing was done using five, seven, nine, and thirteen measurement points for each probe articulation.

Calibration Sphere Error Simulation

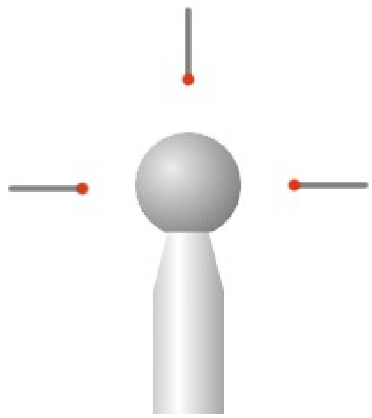

Figure 2: Calibration sphere showing stem and three of the nine test angles.

Probe Angle	*5 Point Pattern*	*7 Point Pattern*	*9 Point Pattern*	*13 Point Pattern*
A0, B0	Five points around circumference, one point on top.	Seven points around circumference, one point on top.	Five points around circumference, three points at 45 degree elevation, one point on top.	Seven points around circumference, five points at 45 degree elevation, one point on top.
A90, B-180	Five points around circumference less 90 degrees of sphere stem, one point normal to probe.	Seven points around circumference less 90 degrees of sphere stem, one point normal to probe.	Five points around circumference less 90 degrees of sphere stem, three points at 45 degree elevation less 90 degrees of sphere stem, one point normal to probe.	Seven points around circumference less 90 degrees of sphere stem, five points at 45 degree elevation less 90 degrees of sphere stem, one point normal to probe.
A90, B-135				
A90, B-90				
A90, B-45				
A90, B0				
A90, B45				
A90, B90				
A90, B135				

Test Parameters

The simulated probe calibration test was performed with the following conditions:

- Sphere form error from 0.000025 mm to 0.000400 mm in steps of 0.000025 mm.
- Measurements repeated 20 times at the given sphere form error level. A new sphere was generated for each of the 20 sets of measurements.
- Sphere diameter of 25 mm.

Calibration Sphere Error Simulation

The range of the form error is in the range of typical sphere form errors from the probe stylus and a CMM calibration sphere. Additional tests were performed with a sphere form error going as high as 0.010 mm but these results are outside of the range of expected sphere form errors and not included in this article.

The test parameters were put into a shell script which contained individual entries with specific parameters and output file names. The following shows parts of the script file:

```
#!/bin/bash
# Generate test data for different levels of form error

# Five Point Sphere
./calsphere -file_data data_5h_0.000025f.txt -file_summary summary_5h_0.000025f.txt -form 0.000025 -
meas_count 20 -point_count 5 -quiet
./calsphere -file_data data_5h_0.000050f.txt -file_summary summary_5h_0.000050f.txt -form 0.000050 -
meas_count 20 -point_count 5 -quiet
./calsphere -file_data data_5h_0.000075f.txt -file_summary summary_5h_0.000075f.txt -form 0.000075 -
meas_count 20 -point_count 5 -quiet
./calsphere -file_data data_5h_0.000100f.txt -file_summary summary_5h_0.000100f.txt -form 0.000100 -
meas_count 20 -point_count 5 -quiet
...

# Thirteen Point Sphere
./calsphere -file_data data_13h_0.000025f.txt -file_summary summary_13h_0.000025f.txt -form 0.000025 -
meas_count 20 -point_count 13 -quiet
./calsphere -file_data data_13h_0.000050f.txt -file_summary summary_13h_0.000050f.txt -form 0.000050 -
meas_count 20 -point_count 13 -quiet
./calsphere -file_data data_13h_0.000075f.txt -file_summary summary_13h_0.000075f.txt -form 0.000075 -
meas_count 20 -point_count 13 -quiet
...
```

Evaluation of Results

The output data includes the raw measurement points for each probe tip calibration and the raw measurement points for the control sphere. The output summary shows the results of the probe calibration for each probe angle and the results of the control sphere. The following shows partial examples of the summary and data files:

Summary File:

```
# Control Sphere: Diameter: 25.00000 Form: 0.00040 Hits: 6
       Location [XYZ]: 0.000000000, 0.000000000, 0.000000000
       Diameter      : 25.000000000
       Form          : 0.000400000
       3D Distance   : 0.000000000
       Tip Diam Err  : 0.000000000
# Measurement 1: Probe: A0.0, B0.0  Hits: 13
       Location [XYZ]: -0.000038493, -0.000036348, 0.000105666
       Diameter      : 24.999925111
       Form          : 0.000354691
       3D Distance   : 0.000118187
       Tip Diam Err  : -0.000074889
# Measurement 2: Probe: A90.0, B-180.0  Hits: 13
       Location [XYZ]: -0.000015691, 0.000216650, 0.000027368
       Diameter      : 25.000196126
       Form          : 0.000225036
       3D Distance   : 0.000218935
       Tip Diam Err  : 0.000196126
# Measurement 3: Probe: A90.0, B-135.0  Hits: 13
       Location [XYZ]: -0.000081538, 0.000121551, 0.000005312
```

```
       Diameter      : 25.000062476
       Form          : 0.000238731
       3D Distance   : 0.000146463
       Tip Diam Err  : 0.000062476
...
```

Data File:

```
# Control Sphere Points [XYZ]: Diameter: 25.00000 Form: 0.00040 Hits: 6
           9.558723593,     1.955314879,    -7.814252814
          -9.558723593,    -1.955314879,     7.814252814
          -1.081867029,    12.327707363,     1.761304896
           1.081867029,   -12.327707363,    -1.761304896
           7.982058212,    -0.670547535,     9.596203036
          -7.982058212,     0.670547535,    -9.596203036
# Measurement 1 Sphere Points [XYZ]: Probe: A0.0, B0.0  Hits: 13
          12.500115451,     0.000040995,    -0.000093167
           7.793726556,     9.772770826,    -0.000095108
          -2.781497398,    12.186404897,    -0.000025432
         -11.262197079,     5.423427524,     0.000063396
         -11.262232653,    -5.423501393,     0.000104485
          -2.781577331,   -12.186423142,     0.000066895
           7.793662455,    -9.772719707,    -0.000021068
           8.838850522,    -0.000004494,     8.838821344
           5.510931020,     6.910359041,     8.838819971
          -1.966881550,     8.617056060,     8.838869240
          -7.963641804,     3.834908897,     8.838932050
          -7.963666959,    -3.835028095,     8.838961104
          -0.000093167,    -0.000047350,    12.500074187
# Measurement 2 Sphere Points [XYZ]: Probe: A90.0, B-180.0  Hits: 13
          -8.838850522,     0.000004494,    -8.838821344
         -12.500115451,    -0.000040995,     0.000093167
          -8.838982279,    -0.000062469,     8.838953101
          -0.000093167,    -0.000047350,    12.500074187
           8.838850522,    -0.000004494,     8.838821344
          12.500115451,     0.000040995,    -0.000093167
           8.838982279,     0.000062469,    -8.838953101
           0.000036891,    -8.838667189,    -8.838853741
          -6.250040130,    -8.838697493,    -6.249957028
          -8.838945388,    -8.838729659,     0.000099360
          -6.250133296,    -8.838744844,     6.250117158
          -0.000094866,    -8.838734153,     8.838920704
          -0.000040995,   -12.499810363,     0.000047350
# Measurement 3 Sphere Points [XYZ]: Probe: A90.0, B-135.0  Hits: 13
          -6.250012344,    -6.249892197,    -8.838816964
          -8.838945388,    -8.838729659,     0.000099360
          -6.250144101,    -6.249959160,     8.838957481
          -0.000093167,    -0.000047350,    12.500074187
           6.250012344,     6.249892197,     8.838816964
           8.838945388,     8.838729659,    -0.000099360
           6.250144101,     6.249959160,    -8.838957481
           6.250103107,    -6.249851202,    -8.838910131
           1.830611117,   -10.669225838,    -6.250010321
          -0.000040995,   -12.499810363,     0.000047350
           1.830517951,   -10.669273188,     6.250063865
           6.249971349,    -6.249918166,     8.838864315
           8.838887413,    -8.838671683,    -0.000032397
...
```

The control sphere is calculated from the six points taken along the randomly generated axis and is expected to show a perfect position and diameter. The form error from the control sphere is expected to show an error identical to the value of the form parameter used to generate the distorted calibration sphere. The control sphere summary and data output was added for validation purposes. When processing the results the *3D Distance* and *Tip Diam Err* fields were used for the analysis.

The *3D Distance* is calculated from the measured XYZ location of the sphere. One advantage of the simulation method over actual testing on a CMM is the known location of the calibration sphere. When testing on a CMM the relative error between the different probe positions can be tested but not an individual probe position.

Using the 20 samples for the *3D Distance* the standard deviation is calculated from all the samples then increased by a factor of 3 to represent 99% of the expected range. The estimated range from the 3 sigma result was compared to the actual range of the measurements in order to validate the method.

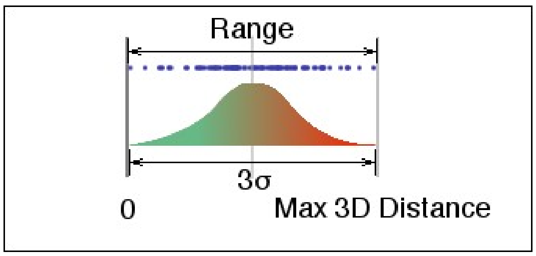

Figure 3: Estimation of the range from 3 Sigma result.

Figure 4, 5, 6, and 7 show the effect of the sphere form error on the position error of the calibrated stylus for 5, 7, 9, and 13 point measurement patterns.

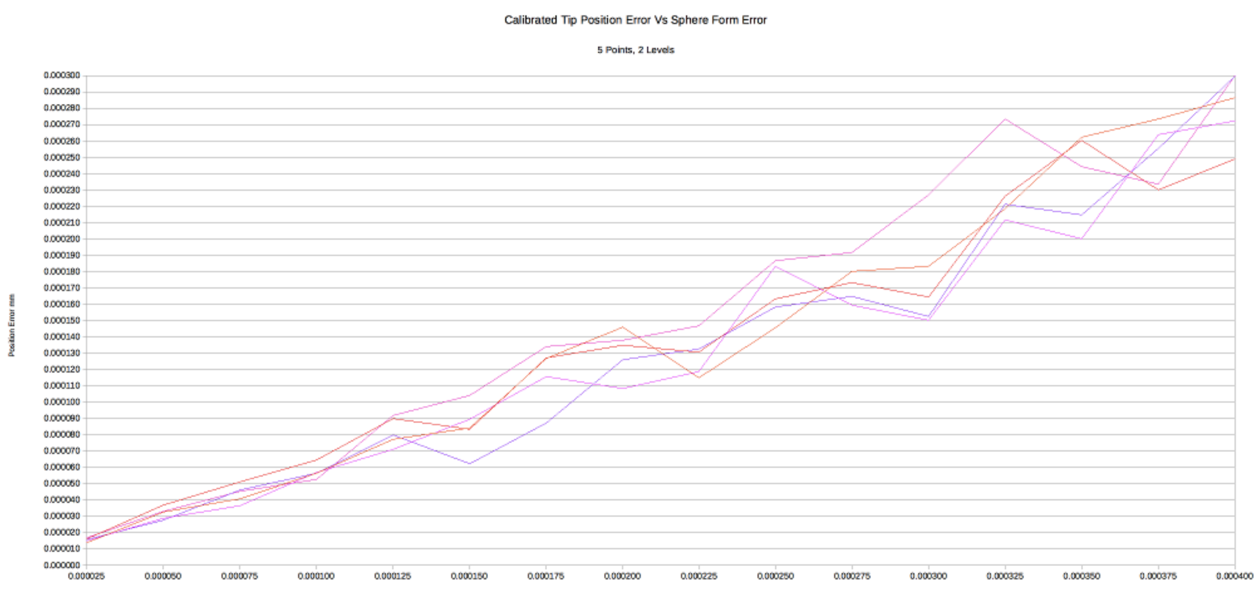

Figure 4: Position error calibrating the probe stylus using 5 points.

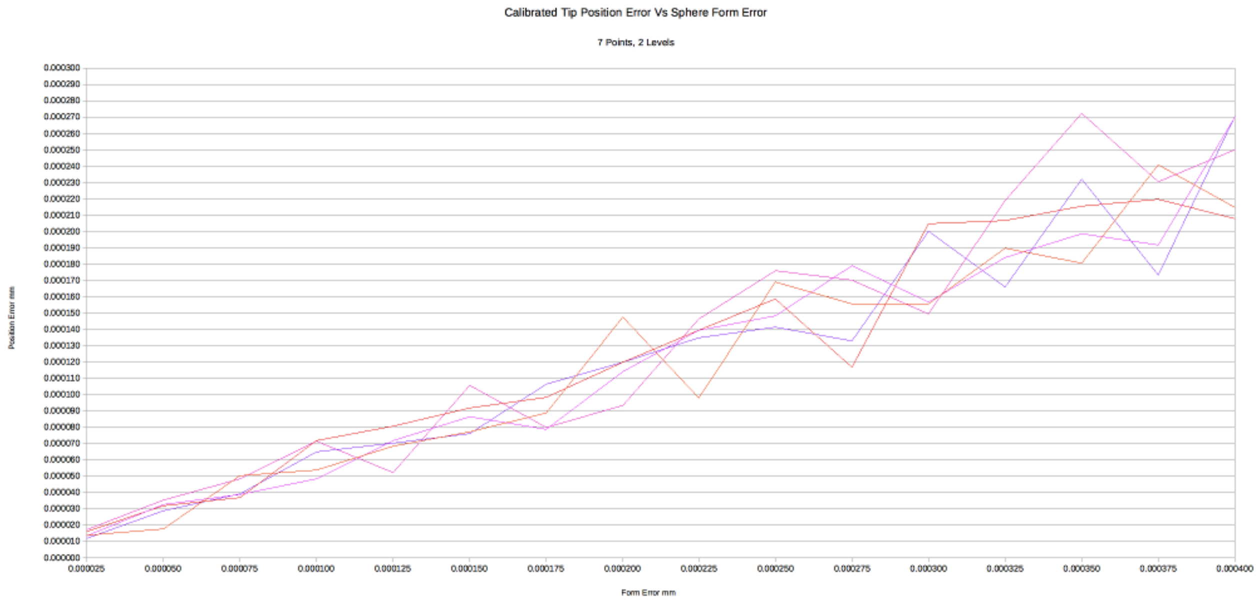

Figure 5: Position error calibrating the probe stylus using 7 points.

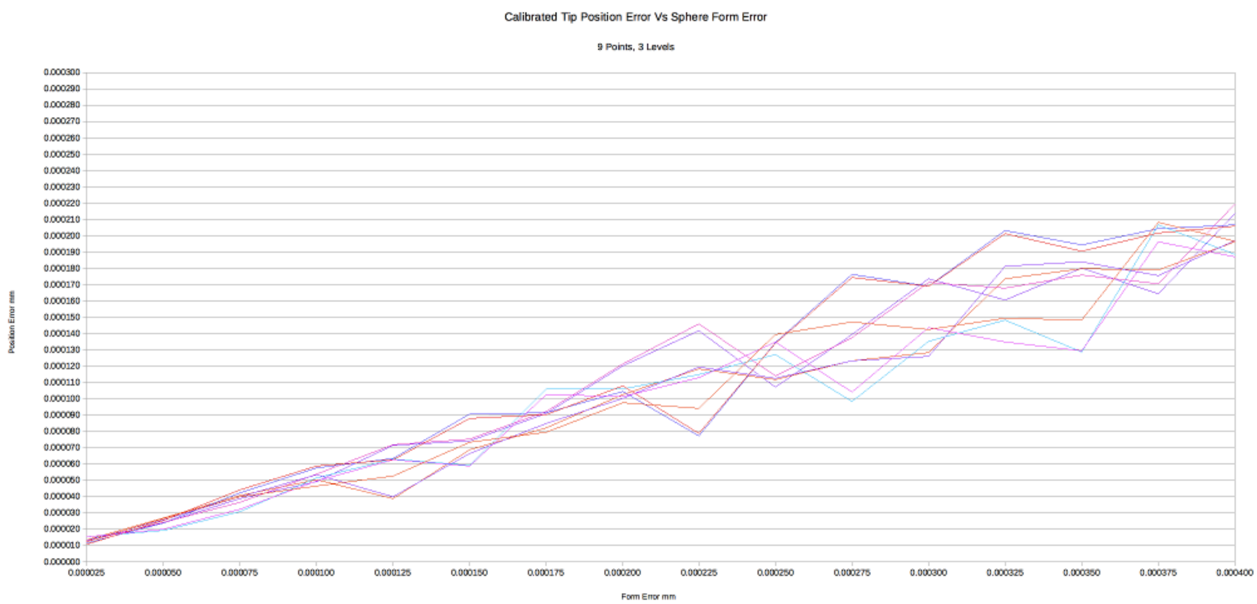

Figure 6: Position error calibrating the probe stylus using 9 points.

Calibration Sphere Error Simulation

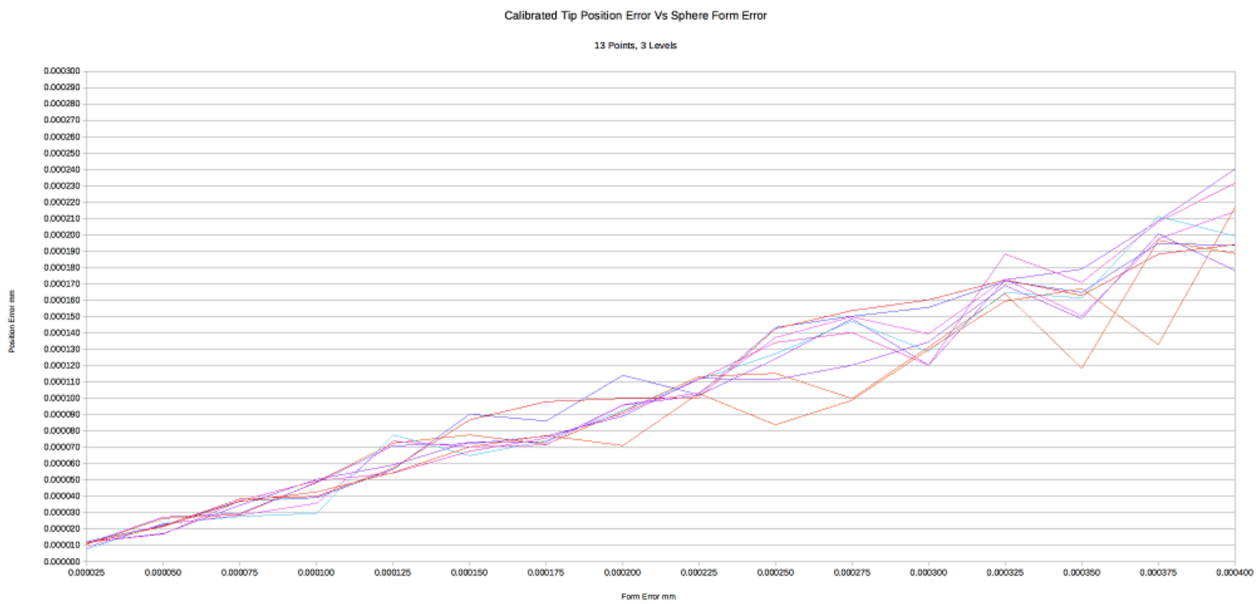

Figure 7: Position error calibrating the probe stylus using 13 points.

The *Tip Diam Err* is calculated from the difference in the measured to nominal calibration sphere diameter. When a probe is calibrated on a CMM the stylus diameter is calculated as the difference in the measured diameter from points at the centre of the probe stylus to the theoretical sphere diameter. The tip diameter for the simulated probe calibration is irrelevant and assumed to be zero so the *Tip Diam Err* average is also zero.

Using the 20 samples for the *Tip Diam Err* the standard deviation is calculated from all the samples then increased by a factor of 3 to represent 99% of the total range and divided by 2 to represent the +/- error from the nominal sphere diameter.

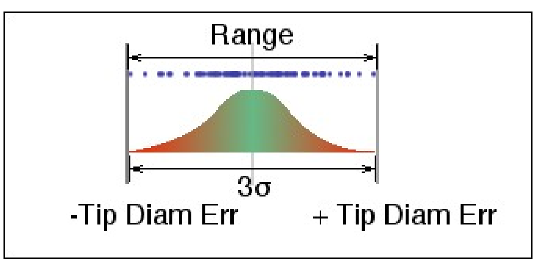

Figure 8: Interpretation of the Tip Diam Error from the 3 Sigma range as a +/- result.

Figure 9, 10, 11, and 12 show the effect of the sphere form error on the size error of the calibrated stylus for 5, 7, 9, and 13 point measurement patterns.

Calibration Sphere Error Simulation

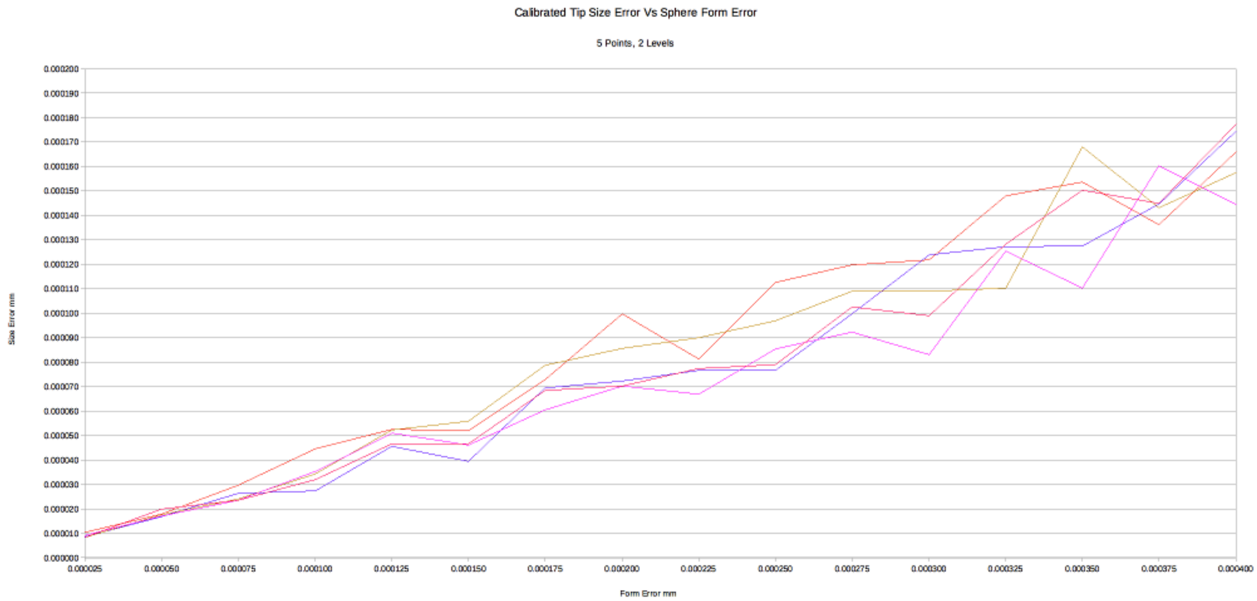

Figure 9: Stylus size error calibrating the probe using 5 points.

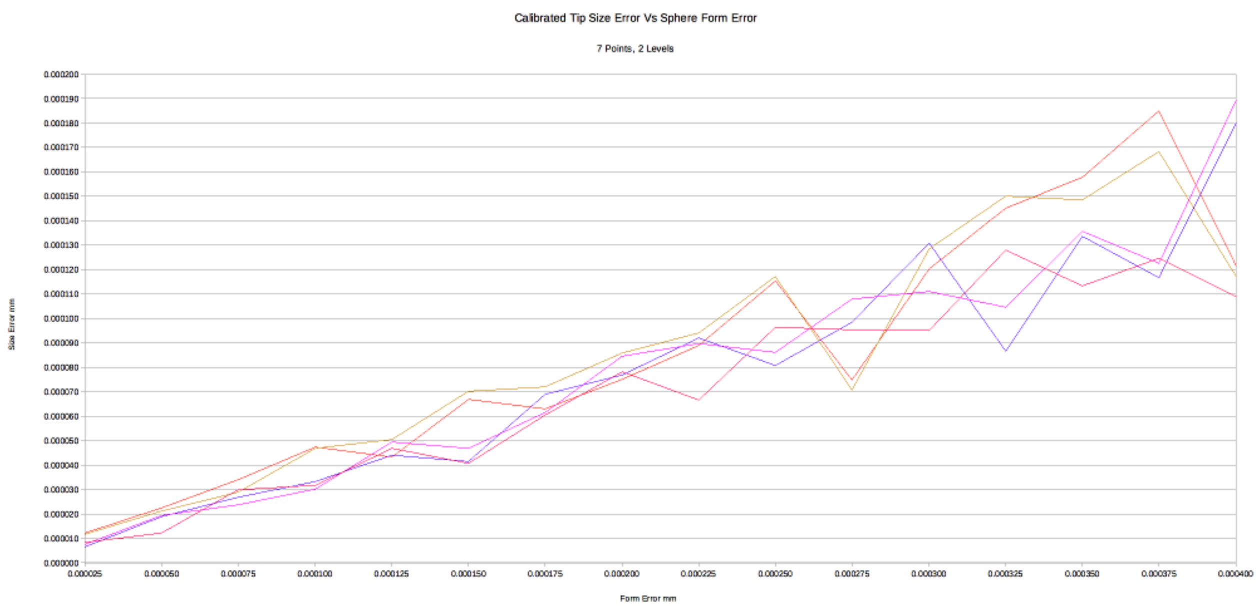

Figure 10: Stylus size error calibrating the probe using 7 points.

Calibration Sphere Error Simulation

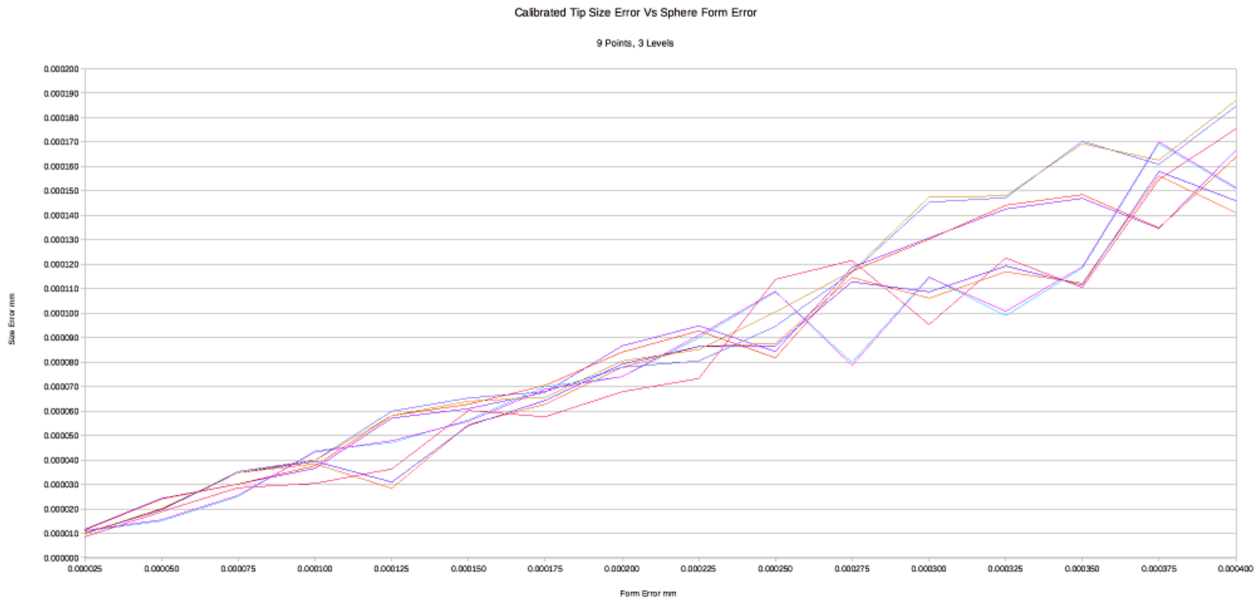

Figure 11: Stylus size error calibrating the probe using 9 points.

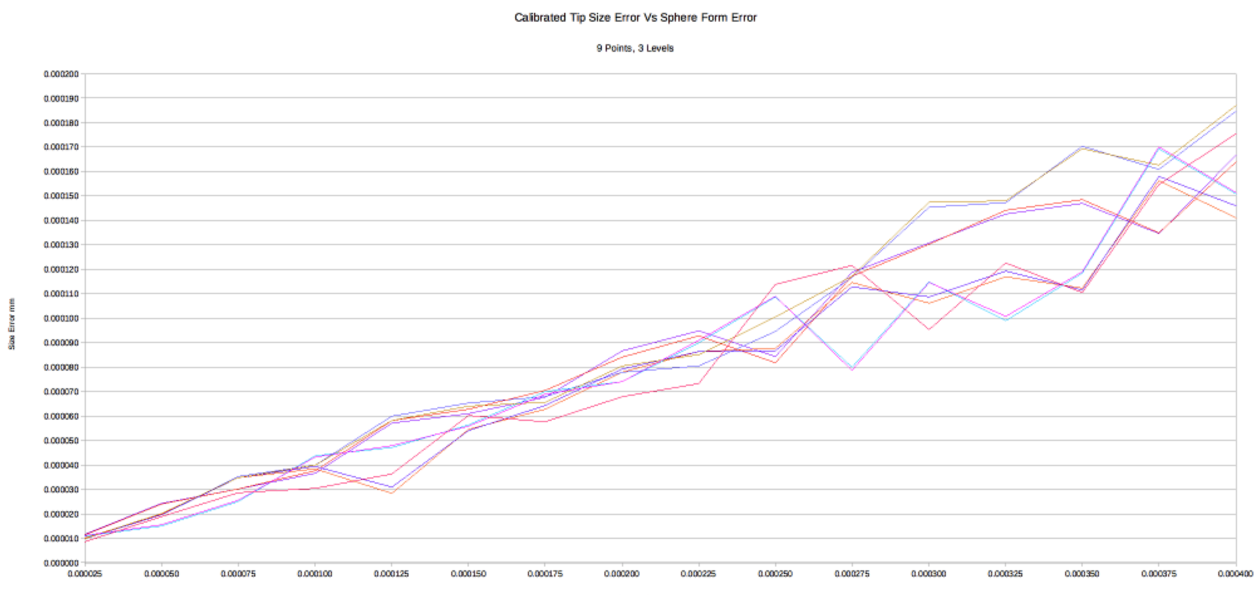

Figure 12: Stylus size error calibrating the probe using 13 points.

Observations

- The magnitude of the position error decreases as the number of points measured on the sphere increases. When calibrating probes using 5 points the ratio of position error to form error is 0.7:1 and drops to 0.5:1 for 13 point sphere measurements.

- The stylus size error does not change by a significant amount as the number of sphere points increase. The error in tip size relative to the sphere form error is 0.4:1.

Compile Notes

The calsphere program is provided as source code. A makefile is included suitable for GNU GCC C++ compilers or equivalent. To create the executable from the source enter the command *make* in a console window from the directory of the source code. The C++ compiler must be C++11 or newer as it uses the newer random number generator features from C++11.

Compile Example:

```
Rons-iMac:calsphere ron$ make clean
rm -rf *.o calsphere calsphere.exe

Rons-iMac:calsphere ron$ make
g++ -c -Wall -c main.cpp
g++ -c -Wall -c vector3.cpp
g++ -c -Wall -c mat4.cpp
g++ -c -Wall -c calsphere.cpp
g++ -c -Wall -c cmdline.cpp
g++ -c -Wall -c bestfitsphere.cpp
g++ main.o vector3.o mat4.o calsphere.o cmdline.o bestfitsphere.o -o calsphere

Rons-iMac:calsphere ron$ ./calsphere -v
Calsphere version 1.0  Ron Jakl 2018
```

Command Line Options

The calsphere program has the following command line options:

```
Rons-iMac:calsphere ron$ ./calsphere -h
Command line options:
           -diameter Sphere diameter
          -file_data Name of output data file
       -file_summary Name of output summary file
               -form Sphere Form Error
                  -h Display argument help and exit
         -meas_count Measurement count
        -point_count Sphere point count
              -quiet Suppress output
                  -v Show version and exit
```

Option	*Description*
-diameter <val>	Nominal size of the calibration sphere used for the simulated measurements.
-file_data <name>	Name of the output file containing all the individual measurement points. This data is intended for independent verification of the results.
-file_summary <name>	Name of the output file containing all of the individual measurement results.
-form <val>	Amount of form error for the calibration sphere.
-h	Display command line option list (help).
-meas_count <val>	Number of times to perform the measurement test. The measurement

Option	Description
	of each pattern of probes is re-ran this amount of times. The output files will contain this number of results for each set of probe positions.
-point_count <val>	The number of points to measure on the calibration sphere.
-quiet	Suppress console output
-v	Show program version and exit.

The calsphere program can be run without parameters and will use default values for all input options.

```
Rons-iMac:calsphere ron$ ./calsphere
INFO:  Generating simulated measurement data ... Done
INFO:  Measurment Count: 10
INFO:  Writing output data file 'data.txt' ... Done
INFO:  Processing complete
INFO:  Writing output summary file 'summary.txt' ... Done
INFO:  Processing complete


default values:

-diameter = 25.0
-file_data = data.txt
-file_summary = summary.txt
-form = 0.001
-meas_count = 10
-point_count = 5
```

Measurement Uncertainty

Based on the simulation results it is possible to estimate the contribution of measurement uncertainty from the probe calibration on a CMM when measuring a length using two different probe positions. The source of error is the product of the calibration sphere form error and stylus form error for each probe position used in the measurement.

A typical calibration sphere is grade 10 (form error of 0.000254 mm) and a typical Renishaw probe stylus is grade 5 (form error of 0.000127 mm). For the estimation of the measurement uncertainty it is assumed that both probes are calibrated using 5 points on the calibration sphere.

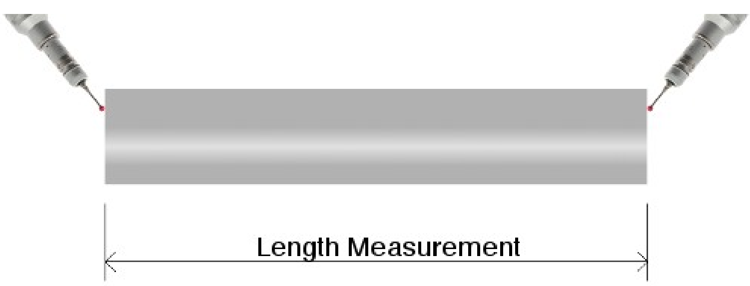

Figure 13: Length measurement example using two different probe positions.

Calibration Sphere Error Simulation

Calibration Sphere Error

Assuming the calibration sphere error is 0.000254 mm (grade 10) and five points are used for the probe calibration the following is the expected errors:

Description	*Value*
Position Error	0.000175 mm
Tip Diameter Error	0.000102 mm
Tip Radius Error	0.000051 mm

These errors are identical for both probe positions. When measuring compensated points the error in the stylus radius is the error of interest and not the full diameter so this value was included in the above list.

Probe Stylus Error

Assuming the probe stylus form error is 0.000127 mm (grade 5) and five points are used for the probe calibration the following is the expected errors:

Description	*Value*
Position Error	0.000087 mm
Tip Diameter Error	0.000051 mm
Tip Radius Error	0.000026 mm

These errors are identical for both probe positions. When measuring compensated points the error in the stylus radius is the error of interest and not the full diameter so this value was included in the above list.

Combined Uncertainty

The measurement uncertainty of length using two probe positions will be the quadrature sum of the different error sources:

Description	*Error Source*	*Value mm*	*Uc Distribution Divisor*	*Standard Uc mm*
Tip1 Position Error	Calibration sphere	0.000175	1.73205	0.000101
Tip1 Position Error	Probe stylus	0.000087	1.73205	0.000050
Tip1 Radius Error	Calibration sphere	0.000051	1.73205	0.000029
Tip1 Radius Error	Probe stylus	0.000026	1.73205	0.000015
Tip2 Position Error	Calibration sphere	0.000175	1.73205	0.000101
Tip2 Position Error	Probe stylus	0.000087	1.73205	0.000050
Tip2 Radius Error	Calibration sphere	0.000051	1.73205	0.000029
Tip2 Radius Error	Probe stylus	0.000026	1.73205	0.000015
			Standard Uncertainty:	0.000166
			Expanded Uncertainty (k=2):	0.000332

Revision History

Date	*Version*	*Changes*
Dec 13, 2018	1.0	Initial release