CMM Uncertainty Budget

Table of Contents

Purpose

This article describes an uncertainty budget generated from a list of potential errors when using a coordinate measuring machine. The estimation of measurement uncertainty is a requirement for ISO 17025 when reporting any measurement value.

Scope

The uncertainty budget in this article is built around the idea of measuring points using a coordinate measuring machine.

The uncertainty budget of a CMM used for measurement can be very complicated if it extends down to individual measurement dimensions. Measuring the distance between two spheres, two points, or two circles can each have a different measurement uncertainty since there are differences in the error sources. There is a large number of combinations that would need to be considered on a case by case basis if uncertainty is required at the level of the dimension.

Consider a simple 3D distance between a plane and a sphere; the final uncertainty is affected by the area of the plane, the relative distance between the plane and the sphere as compared to the area of the plane, the number and pattern of points for each of the features, the form errors of all the features, the type of math used to fit the measured points to the features, uncertainty of the best fit routines and likely other sources. Comparison to definitions of the dimensions types from ASME Y14.5 also complicates things when using a CMM.

One point of view of CMM measurement is that individual measured results are irrelevant provided the end result is functional and fits with the mating part. From this point of view all of the measured features such as cylinders, planes, and spheres are ignored and only the points that were sampled on the part are of interest. The test of an acceptable part would be to verify that all the points fit (or can be forced to fit) inside the tolerance zones described by the GD&T around the nominal shape of the part. It would be similar in principal to using a GO-NOGO gauge to verify the size of a hole where the real size of the hole is not important provided it is somewhere between the upper and lower size limit.

Creating an uncertainty budget around specific measurement dimension types could be done by prior testing and the use of lookup tables. For example, a measured circle with a form error of 0.025 mm might have a potential error in location of some value that was determined by prior testing. A four point circle as compared to an eight point circle should have a different potential error in location even if everything else is comparable. A short cylinder that is used as a datum will introduce a higher potential error than a longer version of the same cylinder.
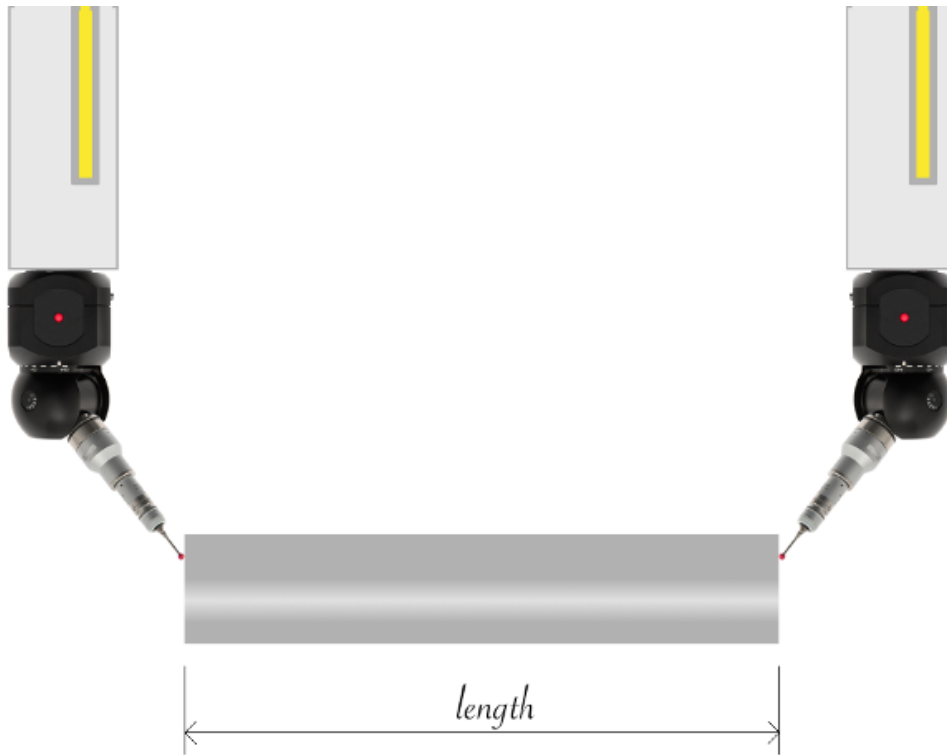
Measurement Example

Illustration 1 shows the measurement that is the basis for the CMM uncertainty budget when using a CMM. The measurement is the length of a block.

The alignment necessary for this measurement is considered part of the measurement and not shown on the illustration.

Illustration 1: Example of length measurement using a CMM.

Uncertainty Groups

The uncertainty budget for a CMM is a collection of individual sources of error. To simplify this the individual error sources are combined into related groups:

- Probe Group

- Calibration Artifact Group

- Environment Group

- CMM Group

- Part Group

Assigning groups of related uncertainty items has some advantages as the individual groups are focused on one particular topic and it is easier to substitute modular items if necessary. Illustration 2 shows an example of a CMM uncertainty budget where the probe and calibration artifacts are treated as modular items. The final uncertainty using different combinations of probes or calibration artifacts can be explored by simply swapping out the different sections of the budget.
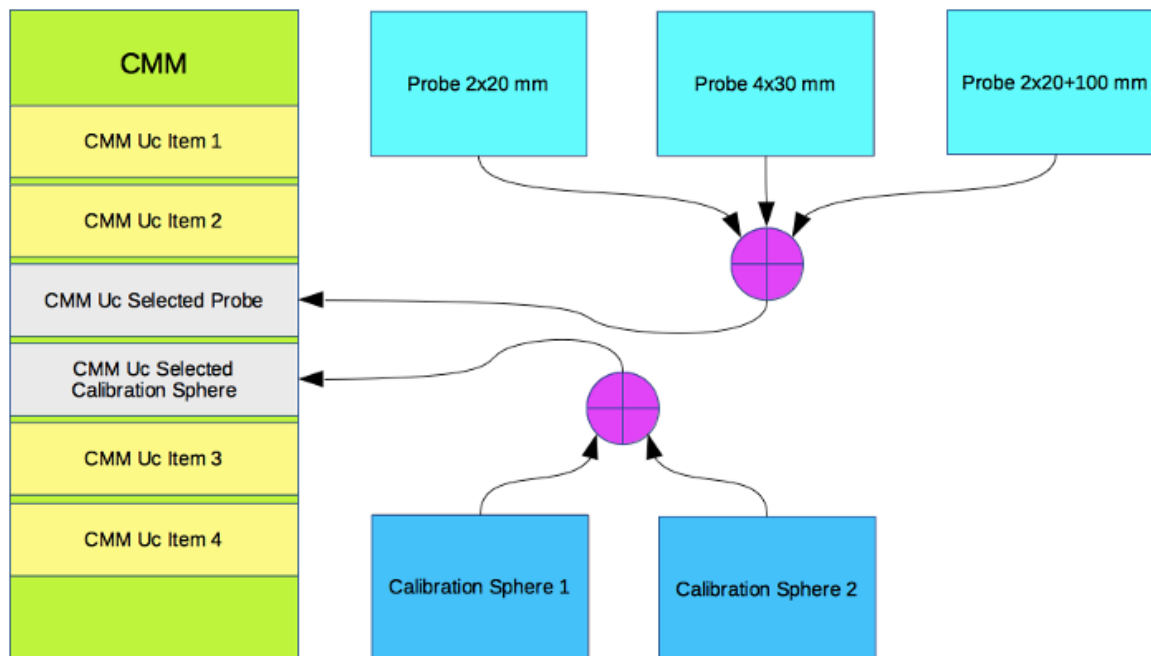
CMM Uncertainty Budget

Illustration 2: Concept for modular selection of probe configuration and calibration sphere.

Probe Group

The probe group describes all the sources of error related to the probing system used on a coordinate measuring machine. Depending on measurement requirements the probe may require relatively long extensions and/or less than optimal stylus configurations. Each configuration of the probing system should therefore be treated separately.

When a CMM is calibrated the errors from the probe are minimized as much as possible so that the real machine error is seen and not obscured by probe errors. Some probe errors will be part of the performance testing of a CMM since a probe is required for testing. The probe type and stylus are usually defined in parallel with the machine specification for this reason.

The following items related to the probe contribute to the measurement uncertainty:

| Item	Error Source Description
Probe Pre-Travel Variation or Probing Error	Probes can contribute a significant amount of measurement error and is affected by the length of the attached stylus. See the *Probe Error* section below.
Probe Tip Calibration Size Variation	The calibrated tip diameter can vary between different probe orientations or even from repeated calibrations. The primary source of this variation is mechanical repeatability of the machine, repeatability of the probe pre-travel variation, and the number of points used for the probe calibration.
Multiple Stylus or Articulation Error	Machines with more than one probe stylus or machines equipped with indexable probe heads can have an error in the relative position of the different stylus tips used for measurements.

The values for the different probe error sources can be determined from tests defined in ISO/IEC 10360-5:2010. It is recommended to use established and recognized tests to determine the errors as opposed to developing custom tests.

Probe Error

The typical touch trigger probe used on a CMM is just an electrical switch. When the stylus is not contacting a surface the switch is closed and opens when the stylus is deflected. In order for a probe to trigger the stylus must deflect by some minimum angle and this is the primary source of the difference between the static and calibrated size of the stylus on a CMM.
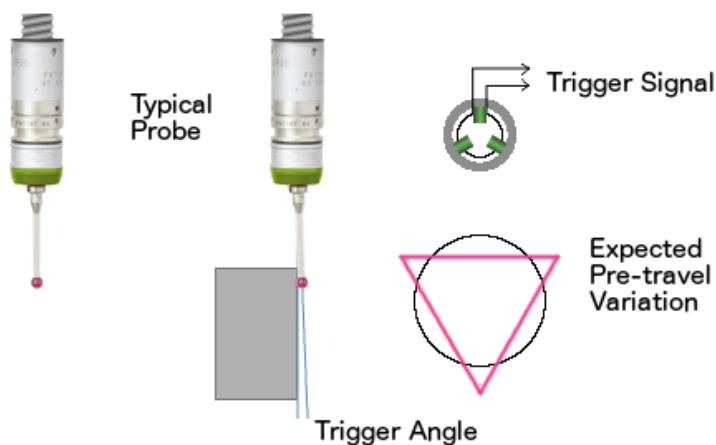
Illustration 3: Typical touch trigger probe

The probe contains three electrical contacts spaced apart 120 degrees to each other. When measuring in a 2D circular pattern the sensitivity of the probe varies based on the trigger direction relative to the probes electrical contacts. This difference in sensitivity of the probe when measuring in a 2D circular pattern appears as a triangular form error when measuring a circular feature. The amount of error from the pre-travel variation depends on the length of the stylus where the longer the stylus the larger the difference between the maximum and minimum variation.

Calibration Artifact Group

The calibration sphere is a critical component when using a CMM. The primary use of the calibration sphere is to allow the software to determine to functional size of the probe tip stylus and relative offsets between different probe tips. Errors in the size of the probe stylus can have a large impact of the measurement of length (bidirectional length measurements in particular). Errors in relative offset will affect measurements that use two or more probe tips. There are different types of calibration artifacts that can be used with a sphere being the simplest and most common on a coordinate measuring machine.

The following items related to the probe calibration artifact contribute to the measurement uncertainty:

CMM Uncertainty Budget

Item	Error Source Description
Calibration Sphere Size Uncertainty	The calibration sphere is used to determine the working diameter of the probe stylus. Errors in the calibration sphere size directly affects the calibrated tip size by a comparable amount. Error in the size of the calibrated probe tip directly contributes to the error in measurement (particularly bidirectional measurements). *It is assumed the actual size of the calibration sphere is used inside the measurement software when calibrating the probe.*
Calibration Sphere Form Error	Errors in the shape of the calibration sphere affect the measured size of the calibrated stylus tip depending on the location of the measured points. The pattern of measurement points when calibrating the probe is usually defined based on the orientation of the probe stylus and the position of the sphere mounting stem.
Calibration Sphere Form Error Uncertainty	The uncertainty of the form error value of the calibration sphere from the laboratory that did the measurement.

Environment Group

The environment of the machine, particularly temperature, is a big factor in the uncertainty of measurement. All measurements are expected to be performed at the nominal reference temperature of 20˚C and if not the results must be adjusted to 20˚C. All material expands and contracts from changes in temperature so this is a key source of measurement error when using a CMM.

The items related to temperature that contribute to measurement error are:

- Expansion or contraction of the machine scales.
- Expansion or contraction of the measured artifact.

How well the temperature of the environment is maintained needs to be determined in order to properly estimate the amount of error. The measurement of temperature includes error which much be considered when estimating the measurement uncertainty from temperature.

The following error sources are related to temperature:

- Uncertainty of the expansion coefficient for the part and CMM scales. The expansion coefficient for common materials is generally known and assumed correct to +/- 10%.
- Difference in the expansion coefficient between the part and the machine scales. If the axis of the coordinate measuring machine happened to have an identical expansion coefficient to the part then no measurement error would be observed.
- Uncertainty of temperature measurement. The temperature measurement may have a bias or other errors where the real temperature (or absolute deviation from the reference temperature) is larger than what is shown by the thermometer.

The following items related to the environment contribute to the measurement uncertainty:

Item	Error Source Description
Environment Temperature	The reference temperature for all measurements is 20˚C. If the

CMM Uncertainty Budget

Item	Error Source Description
Variation	laboratory has an environment that is controlled between 19-21˚C then the variation at any given time could be as high as 1˚C from the reference temperature of 20˚C.
Thermometer Accuracy	The strict acceptance limit or accuracy specification of the thermometer determines the reliability of temperature measurement. If the environment is controlled between 19-21˚C but the thermometer has a potential error of 0.5˚C then the real environment could be anywhere between 18.5-21.5˚C. See section *Thermometer Accuracy* section below.
Thermometer Resolution	A typical thermometer has a resolution of 0.1˚C so the fractional part of the display resolution cannot be used. A thermometer with a display precision of one decimal place means the real temperature could be +/- 0.05˚C from the displayed value.

For this example budget the temperature compensation option in the inspection software is not used (or not available). If temperature compensation is used the uncertainty from the environment is reduced to the uncertainty of temperature measurement and other subtle effects to the coordinate measuring machine related to temperature.

Thermometer Accuracy

The strict acceptance limit represents the value that contains all measured errors increased by the expanded uncertainty. This is the minimum value that contains all the thermometer errors stated with a confidence level of at least 95%.

The *Thermometer Accuracy* is the working tolerance for the thermometer. This value is the higher of the manufacturers specification or the strict acceptance limit. If, for example, a thermometer has an uncorrected measurement error of +/- 0.25˚C and the expanded uncertainty from the calibration laboratory is 0.35˚C then the accuracy specification would be +/- 0.6˚C when using this instrument.

Uncorrected measurement errors in any instrument can be externally removed by various methods provided the error is constant.

CMM Group

A CMM, in its simplest form, is a machine consisting of three orthogonal axis with scales and a probe. The accuracy and stability of the bearing guide ways partly define the volumetric accuracy of the coordinate measuring machine. The expansion coefficient of the three axis scales in the measurement environment along with the capability of the probing system contribute to measurement error.

The coordinate measuring machine has an accuracy specification that should describe how a particular CMM will perform in a specific environment over a long period of time. This specification is partially based on the recommended calibration cycle and proper use of the equipment.

The following items related to the CMM contribute to the measurement uncertainty:

CMM Uncertainty Budget

Item	Error Source Description
CMM Expansion Coefficient Error	The expansion coefficient of the machines axis are generally known but may be off by as much as 10% of the expected value unless the expansion coefficient has been certified.
CMM Scale Resolution Error	A typical CMM scale has a resolution of 0.001 mm. The actual position of the machine could be as much as +/- 0.0005 mm from the reported axis position in this case.
CMM Accuracy	The strict acceptance limit or accuracy specification for the performance of a CMM, whichever is higher. See section *CMM Accuracy* below.

CMM Accuracy

The CMM accuracy is the working tolerance for the machine. This value is the higher of the machine specification or the strict acceptance limit.
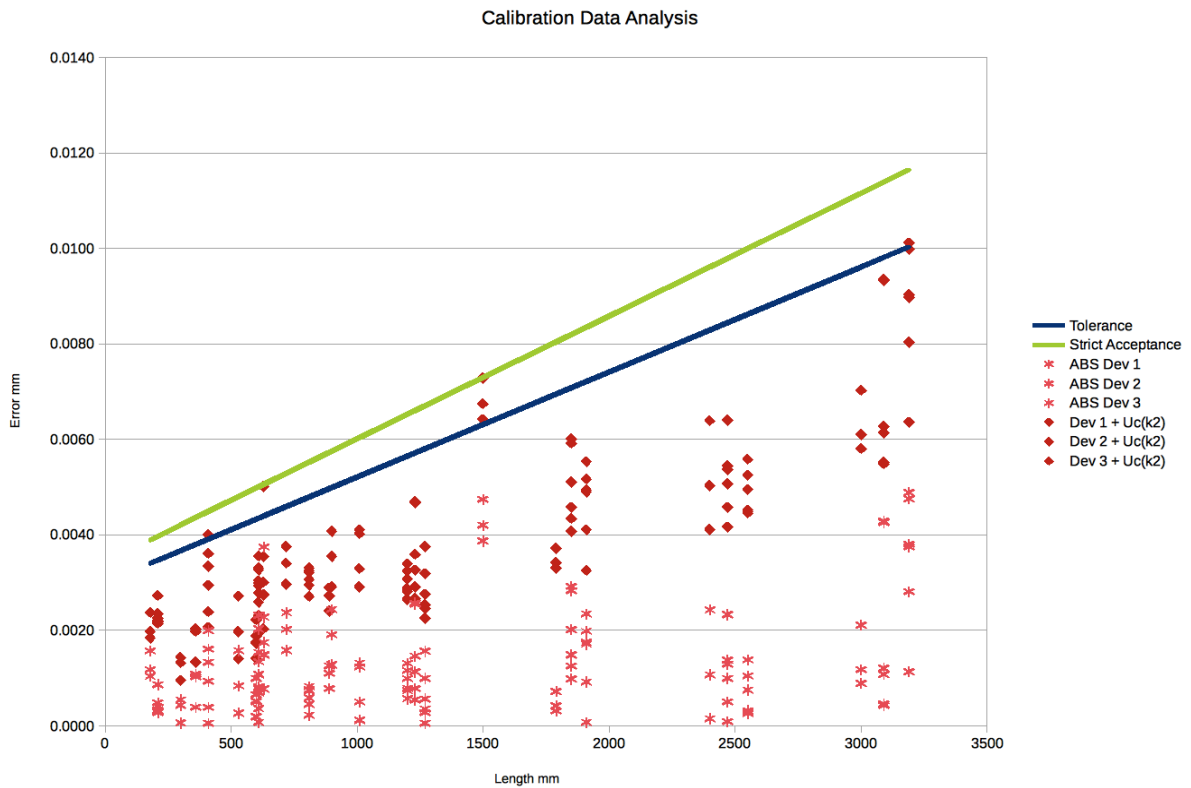
Illustration 4: Analysis of CMM performance test results from ISO 10360-2:2009 to determine the strict acceptance limit.

The strict acceptance limit represents the value that contains all measured errors increased by the expanded uncertainty. This is the minimum value that contains all the machine errors stated with a confidence level of at least 95%.

The calibration of the machine is done in a relatively short period of time so changes in the

July 29, 2018

environment over a 24hr period are rarely seen during a typical calibration. For example, the calibration of a three axis CMM could be completed in as little as 4 hours and during this time the environment would (or should) be relatively stable. Over a period of 24 hrs the environment of a machine is almost certainty going to change more than what was observed in the 4 hour window during the calibration. For this reason it should always be possible to meet or exceed manufacturers specifications when calibrating the machine provided the machine specification takes into account the allowed temperature range and the effect on the CMM. The effect on a CMM from changes in temperature include the geometry of the machine and not just the scales.

For this reason the measurement uncertainty used for the uncertainty budget is the higher of the specification or the strict acceptance limit. If the machine cannot be made to perform within the manufacturers specification reduced by the expanded uncertainty then the measurement uncertainty of the machine should be increased accordingly.

There are valid reasons why the strict acceptance limit could be higher than the machine specification other than problems during calibration. Testing that produces a relatively high measurement uncertainty is one reason or if the specification of the machine is too small (incorrect) this would be another reason.

Using data from the older ASME B89.4.1:1997 ball bar standards to determine the strict acceptance limit can be very difficult (maybe impossible). The newer ASME B89.4.10360-2:2008 or ISO/IEC 10360-2:2009 standards are more suitable for this purpose.

Temperature has an effect on the shape of a coordinate measuring machine that is nearly impossible to estimate without extensive testing in a suitable environment. Many shop floor machines actively compensate for expected changes in the shape of a CMM where the coefficients for the correction are determined by direct testing in an environment chamber.

One error related to scale resolution that is not included in this budget is from scale interpolation. This error is dependent on the quality of the encoder signal, scale irregularities, and quality of the interpolator electronics. It has been observed that high accuracy coordinate measuring machines usually have an extremely precise scale interpolator.

Part Group

The material of the part that is measured on the CMM will be a source of error if the environment is not ideal. If the part material has an expansion coefficient that is the same as the CMM axis then no error is expected as both the part and the machine will grow by the same amount. If the part expands at a rate different than the CMM axis then errors will be observable if the environment temperature is not 20˚C.

The following items related to the part contribute to the measurement uncertainty:

Item	Error Source Description
Part Expansion Coefficient Error	The expansion coefficient of the part can be determined from published sources but may be off by as much as 10% unless the expansion coefficient has been certified.
Part to CMM Expansion Coefficient Difference	The difference in the expansion coefficient of the part to that of the CMM. See section *Expansion Difference* below.

July 29, 2018

CMM Uncertainty Budget

Expansion Difference

The *Expansion Difference* error occurs when the environment is not 20°C and temperature compensation is not used by the inspection software. For example, if measuring an aluminum part on a steel CMM and the temperature happens to be 21°C the expected measurement error would be 0.010 mm/m. This assumes the CMM expansion coefficient is 0.012 mm/m and the part expansion coefficient is 0.022 mm/m resulting in a difference of 0.010 mm/m/°C between the two.
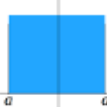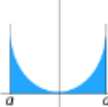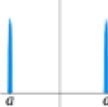
The Expansion Difference source of error can be removed by proper use of temperature compensation. This particular source of error was put into Part Group but could also be in the CMM Group (but not both at the same time).

Temperature has an effect on the shape of part that is nearly impossible to estimate. It is very rare that an odd shaped part, particularly if constructed from different types of materials, will grow or shrink with no change in shape.

Conversion of Potential Error to Standard Uncertainty

Potential errors are not guaranteed errors. The goal of the uncertainty budget is to produce a statistical estimate of the measurement error to a specific confidence level (usually 95%) or, to describe it another way, for 95 out of 100 measurements the real value will be the reported value within a tolerance of the expanded uncertainty.

The standard uncertainty is equivalent to a standard deviation from a normal distribution of data. To convert a potential error to a standard uncertainty, when only knowing minimal details about the error, a standard divisor can be used based on the expected distribution shape. if direct measurement is possible then the standard uncertainty would be the standard deviation of the samples of error. For example, if it was determined that the contribution of temperature on the expansion coefficient of some material had a potential error of +/- 0.0100 mm then the standard uncertainty for this error would be 0.0058 mm (treated as a rectangular distribution since the real value could be anywhere within the expected range).

	Normal	Triangular	Rectangular	U	Step
Graphical					
Equivalent Standard Uncertainty	$S = \sigma$	$s = \dfrac{a}{(\sqrt{6})} \approx 0.41a$	$s = \dfrac{a}{(\sqrt{3})} \approx 0.58a$	$s = \dfrac{a}{(\sqrt{2})} \approx 0.71a$	$s = \dfrac{a}{(2\sqrt{3})} \approx 0.29a$

Illustration 5: Divisors used to convert an error range to a standard uncertainty.

Uncertainty values from a calibration certificate are usually reported with an expanded uncertainty ($k=2$). To convert these to a standard uncertainty the value is divided by the same multiplier used to create the expanded uncertainty (usually 2). The multiplier used to convert the standard uncertainty into an expanded uncertainty is based on two things; the desired confidence level and the effective degrees of freedom from the combined uncertainty data. The effective degrees of freedom takes into account non-normal distributions (t-distributions) that are associated with smaller sample sets. For example, if the effective degrees of freedom of the measurement uncertainty was determined to be 3 then a multiplier of 3.182 would be required to produce a confidence level of 95% whereas if the effective degrees of freedom was found to be 60 then a

July 29, 2018

multiplier of 2.000 would be required to achieve the same 95% confidence level. This is an analogy to comparing a measurement repeated a minimum number of times to one that was performed many times. The smaller sample would have less variation than the larger but, if the effective degrees of freedom was properly taken into account, the value representing the target confidence level would be the same since each would have a different multiplier value.

The combined uncertainty from all the individual standard uncertainties is the quadrature sum of all the individual uncertainty components.

$$Uc_{combined} = Uc_a^2 + Uc_b^2 + Uc_c^2 + Uc_d^2 + ...$$

Uncertainty Budget Entries

When creating an uncertainty budget the components, logic, and calculation should be documented. When questions arise or revisions are necessary having all the details available will make the process easier.

An example of the uncertainty budget based on the items listed in the uncertainty groups is shown in the following sections. Default values and distribution types are indicated in the tables.

Probe Data

Uncertainty Item	Type	Default Value	Type	Distribution	Sensitivity	Comments
Probe – 4x20 mm TP20 Standard Force Module						
Articulation Error	Constant	0.0035	B	Rectangular	1.0	Articulation error of motorized probe head.
Probing Error	Constant	0.0048	B	Rectangular	1.0	Probing error of measurement sensor.
Size Error	Constant	0.0025	B	Rectangular	1.0	Probing error of measurement sensor.

Probe Data Notes

- Probe articulation error value based on ISO/IEC 10360-5 P_{LTE} test.
- Probing error value based on ISO/IEC 10360-5 P_{FTU} test.
- Size error value based on ISO/IEC 10360-5 P_{STE} test.

CMM Uncertainty Budget

Calibration Artifact Data

Uncertainty Item	Type	Default Value	Type	Distribution	Sensitivity	Comments
Calibration Artifact – SN 90210, 25.4 mm Ceramic Sphere						
Form Error	Constant	0.00013	B	Rectangular	1.0	Calibration sphere form error.
Form Uc	Constant	0.00003	B	Normal	1.0	Calibration sphere form error standard uncertainty.
Size Uc	Constant	0.00021	B	Normal	1.0	Calibration sphere size standard uncertainty.

Calibration Artifact Notes

- The sphere form error is from the calibration certificate.
- The sphere form standard uncertainty is from the calibration certificate.
- The sphere size standard uncertainty is from the calibration certificate.

Environment Data

Uncertainty Item	Type	Default Value	Type	Distribution	Sensitivity	Comments
Thermometer – SN T1010 and probe SN P2020						
Max Temperature Variation	Constant	1.0 C	B	U	1.0	Uncertainty from the environment and part expansion coefficient.
Thermometer Accuracy	Constant	0.5 C	B	Rectangular	1.0	Strict acceptance limit of thermometer.
Thermometer Resolution	Constant	0.1 C	B	Step	1.0	Resolution of the thermometer.

Environment Data Notes

- The *Maximum Temperature Variation* represents the maximum deviation in temperature from the reference temperature of 20˚C. A value of 1.0˚C means the environment could be anywhere between 19˚C and 21˚C.
- The distribution type 'U' is more appropriate than a rectangular distribution for the *Max Temperature Variation* entry. The error due to temperature at the nominal of 20˚C would be zero and would increase to the maximum at +/- 1˚C deviation from 20˚C forming a 'U' shape to the data.

July 29, 2018

CMM Uncertainty Budget

CMM Data

Uncertainty Item	Type	Default Value	Type	Distribution	Sensitivity	Comments
CMM – DEA Omega SN 001						
Calibration Sphere Uc	Constant	CALSPH_uc	B	Normal	1.0	Product of the uncertainty from the calibration sphere group.
Expansion Coefficient Uc	Length Dependent	ENV_uc * CTEm * 0.1	B	Normal	1.0	Uncertainty of the machines expansion coefficient (10%)
Probe Uc	Constant	PROBE_uc	B	Normal	1.0	Product of the uncertainty from the probe group.
Scale Resolution	Constant	SR	B	Step	1.0	Uncertainty from the scale resolution
Specification	Constant	0.0034+0.0026*L/1000	B	Rectangular	1.0	Strict acceptance limit from testing. The expression contains the constant and length dependent components.

CMM Data Notes

- Expansion coefficient of the machines axis is assumed to have an error no greater than 10% of the nominal expansion coefficient. For example, if the nominal expansion coefficient for the axis is 0.010 mm/m/°C then uncertainty would be +/- 0.001 mm/m/°C.
- The *Expansion Coefficient Uc* item is already a standard uncertainty therefore the divisor is set to normal.

CMM Data Variables

Variable Name	Description
CALSPH_uc	Product of the uncertainty from the calibration sphere.
ENV_uc	Product of the uncertainty from the environment.
CTEm	Expansion coefficient of the machine (CMM).
PROBE_uc	Product of the uncertainty from the probe configuration used on the machine.
SR	Scale resolution
L	Test length.

CMM Uncertainty Budget

Inspected Part Data

Uncertainty Item	Type	Default Value	Type	Distribution	Sensitivity	Comments
Inspection Part						
Expansion Coefficient Uc	Length Dependent	ENV_uc * CTEc * 0.1	B	Normal	1.0	Uncertainty of the part expansion coefficient (10%)
Expansion Coefficient Difference	Constant	ENV_uc * CTEd	B	Normal	1.0	Uncertainty from the difference between the CMM and the part expansion coefficient.

Inspected Part Data Notes

- Expansion coefficient of the part is assumed to have an error no greater than 10% of the nominal value. For example, if the nominal expansion coefficient for the part is 0.022 mm/m/˚C then the uncertainty would be +/- 0.0022 mm/m/˚C.
- The *Expansion Coefficient Difference* is the error from the difference in the expansion coefficient between the part and the axis of the CMM.
- The *Expansion Coefficient Uc* and *Expansion Coefficient Difference* is already a standard uncertainty therefore the divisor is set to normal.

Inspected Part Data Variables

Variable Name	Description
ENV_uc	Product of the uncertainty from the environment.
CTEc	Expansion coefficient of the part.
CTEd	Absolute difference between the expansion coefficient of the machine (CMM) and the part.

Sample Budget

The uncertainty budget data was entered into the Uc Budget Editor program and produced the following output:

CMM Uncertainty Budget

Budget Definition - CMM Measurement Uncertainty

Environment Component T1010_P2020 Variable

Source	Value	Expression	Type	Sensitivity	Distribution	Distribution Divisor
Max Temperature Variation	Constant	1.0	Type B	1.000000	U	1.414214
Thermometer Accuracy	Constant	0.5	Type B	1.000000	Rectangular	1.732051
Thermometer Resolution	Constant	0.1	Type B	1.000000	Step	3.464102

Probe Component Probe 4x20 Variable

Source	Value	Expression	Type	Sensitivity	Distribution	Distribution Divisor
Probing Error	Constant	0.0048	Type B	1.000000	Rectangular	1.732051
Articulation Error	Constant	0.0035	Type B	1.000000	Rectangular	1.732051
Size Error	Constant	0.0025	Type B	1.000000	Rectangular	1.732051

Calibration Sphere Component SPH 90210 Variable

Source	Value	Expression	Type	Sensitivity	Distribution	Distribution Divisor
Form Error	Constant	0.00013	Type B	1.000000	Rectangular	1.732051
Size Uc	Constant	0.00021	Type B	1.000000	Normal	1.000000
Form Uc	Constant	0.00003	Type B	1.000000	Normal	1.000000

CMM Component Omega 001

Source	Value	Expression	Type	Sensitivity	Distribution	Distribution Divisor
Expansion Coefficient Uc	Length Dependent	ENV_uc*CTEm*0.1	Type B	1.000000	Normal	1.000000
Scale Resolution	Constant	Sr	Type B	1.000000	Step	3.464102
Probe Uc	Constant	PROBE_uc	Type B	1.000000	Normal	1.000000
Specification	Constant	0.0034+0.0026*L/1000	Type B	1.000000	Rectangular	1.732051
Calibration Sphere Uc	Constant	CALSPH_uc	Type B	1.000000	Normal	1.000000

Inspected Part Component

Source	Value	Expression	Type	Sensitivity	Distribution	Distribution Divisor
Expansion Coefficient Uc	Length Dependent	ENV_uc * CTEc * 0.1	Type B	1.000000	Normal	1.000000
Expansion Difference	Length Dependent	ENV_uc * CTEd	Type B	1.000000	Normal	1.000000

Global Variables

CTEm	0.010000
Sr	0.001000
Td	0.000000
Tr	2.000000
Sdev	0.000000

Illustration 6: First page of budget report from the Uc Budget Editor program. This page lists all the components and expressions without values.

Budget Data - CMM Measurement Uncertainty

	Position (mm)	0.0	250.0	500.0	750.0	1000.0	1250.0	1500.0	1750.0	2000.0	2250.0	2500.0
Environment Component T1010_P2020 Variable												
Export Variables	ENV_uc	0.76431	0.76431	0.76431	0.76431	0.76431	0.76431	0.76431	0.76431	0.76431	0.76431	0.76431
Source	Max Temperature Variation	0.70711	0.70711	0.70711	0.70711	0.70711	0.70711	0.70711	0.70711	0.70711	0.70711	0.70711
	Thermometer Accuracy	0.28868	0.28868	0.28868	0.28868	0.28868	0.28868	0.28868	0.28868	0.28868	0.28868	0.28868
	Thermometer Resolution	0.02887	0.02887	0.02887	0.02887	0.02887	0.02887	0.02887	0.02887	0.02887	0.02887	0.02887
Probe Component Probe 4x20 Variable												
Export Variables	PROBE_uc	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372
Source	Probing Error	0.00277	0.00277	0.00277	0.00277	0.00277	0.00277	0.00277	0.00277	0.00277	0.00277	0.00277
	Articulation Error	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202	0.00202
	Size Error	0.00144	0.00144	0.00144	0.00144	0.00144	0.00144	0.00144	0.00144	0.00144	0.00144	0.00144
Calibration Sphere Component SPH 90210 Variable												
Export Variables	CALSPH_uc	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023
Source	Form Error	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008
	Size Uc	0.00021	0.00021	0.00021	0.00021	0.00021	0.00021	0.00021	0.00021	0.00021	0.00021	0.00021
	Form Uc	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003
CMM Component Omega 001												
Local Variables	L	0.00000	250.000	500.000	750.000	1000.00	1250.00	1500.00	1750.00	2000.00	2250.00	2500.00
Source	Expansion Coefficient Uc	0.00000	0.00019	0.00038	0.00057	0.00076	0.00096	0.00115	0.00134	0.00153	0.00172	0.00191
	Scale Resolution	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029	0.00029
	Probe Uc	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372	0.00372
	Specification	0.00196	0.00234	0.00271	0.00309	0.00346	0.00384	0.00421	0.00459	0.00497	0.00534	0.00572
	Calibration Sphere Uc	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023	0.00023
Inspected Part Component												
Local Variables	CTEd	0.00200	0.00200	0.00200	0.00200	0.00200	0.00200	0.00200	0.00200	0.00200	0.00200	0.00200
	CTEc	0.01200	0.01200	0.01200	0.01200	0.01200	0.01200	0.01200	0.01200	0.01200	0.01200	0.01200
Source	Expansion Coefficient Uc	0.00000	0.00023	0.00046	0.00069	0.00092	0.00115	0.00138	0.00161	0.00183	0.00206	0.00229
	Expansion Difference	0.00000	0.00038	0.00076	0.00115	0.00153	0.00191	0.00229	0.00268	0.00306	0.00344	0.00382
Uncertainty												
	Combined	0.00422	0.00444	0.00472	0.00506	0.00545	0.00588	0.00634	0.00682	0.00733	0.00785	0.00838

Uncertainty Expression

$Uc(K=1): 0.00418+0.00103L/1000+0.00027(L/1000)^2$
$Uc(K=2): 0.00835+0.00205L/1000+0.00053(L/1000)^2$

Illustration 7: Second page of the report from the Uc Budget Editor program. This page shows the calculated values of each uncertainty item at different lengths through the measurement volume of the CMM.
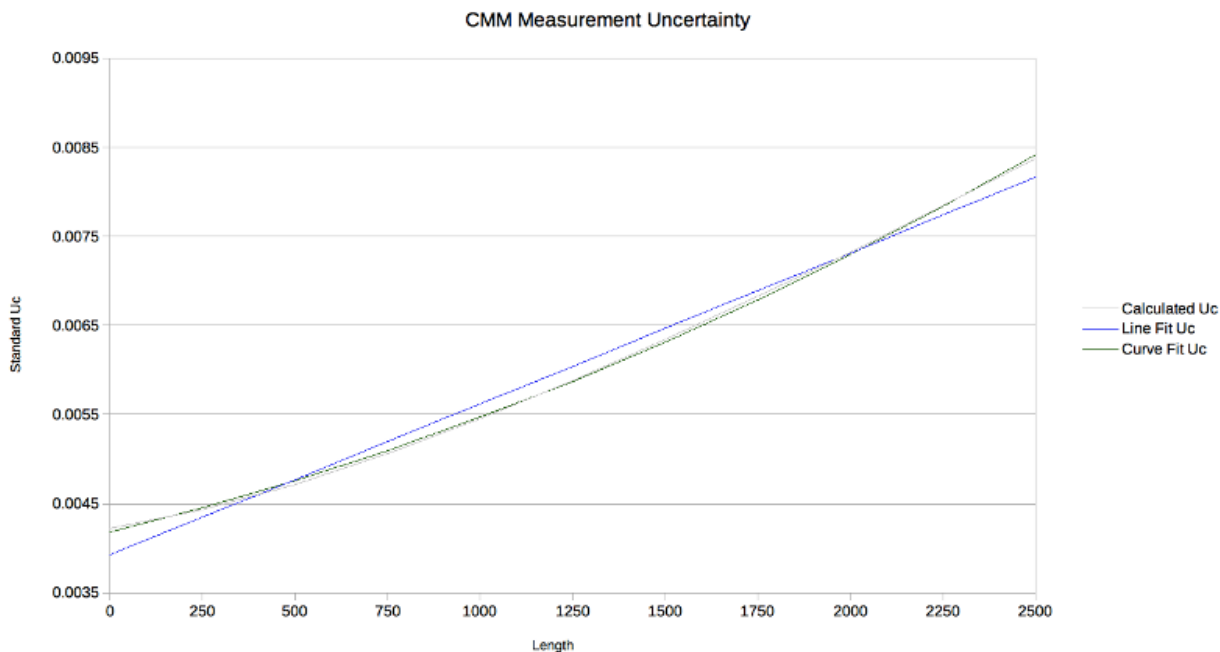
July 29, 2018

Illustration 8: Analysis of the standard uncertainty over a length of 2500 mm.

Uncertainty Expression

When describing the uncertainty an expression is often used since the measurement length is not fixed. If, for example, the CMM only measured parts that were exactly 1025 mm long then a single uncertainty value at the specified length would be appropriate but this is rarely true.

One method to express measurement uncertainty is with a best fit line through the uncertainty data samples from each test length. Using the data from the sample uncertainty budget and fitting a line through the samples the uncertainty expression would be:

$$Uc = 0.0039 + 0.0017\,L\,mm$$

$$Uc(k2) = 0.0078 + 0.0034\,L\,mm$$

Where L is in meters

One problem with using a best fit line is that the uncertainty shape is always curved in such a way that the zero length uncertainty is always less than the results calculated directly from the budget data. An example of this can be seen in illustration 8 where the actual zero length uncertainty is significantly higher than the uncertainty calculated at length zero from the best fit line. If the curve is pronounced enough the first term could be zero or even negative (which is not possible for measurement uncertainty).

One work around to this problem is to describe the uncertainty using only the first and last test length values. The result of from this method using the same data would be:

$$Uc = 0.0042 + 0.0017\,L\,mm$$

$$Uc(k2)=0.0084+0.0034\,L\,mm$$

Where L is in meters

Another solution to the best fit line problem is to describe the uncertainty as a curve. An example of a curve fit in comparison to the actual uncertainty data can be seen in illustration 8. Using the data from the sample uncertainty budget the uncertainty expressions using a curve would be:

$$Uc=0.0042+0.0010\,L+0.0003\,L^2\,mm$$

$$Uc(k2)=0.0084+0.0020\,L+0.0005\,L^2\,mm$$

Where L is in meters

Revision History

Revision	Date	Reason
1	Jan 8, 2018	Initial Release
2	July 29, 2018	Correction of thermometer strict acceptance example.

July 29, 2018